Fast B-Spline 2D Curve Fitting for unorganized Noisy Datasets

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Context

1. Optical and Tactile Metrology for Absolute Form Characterization (EURAMET project IND10)
2. Fast polynomial spline curve reconstruction from very large unstructured datasets

Objective

Curve reconstruction of freeform shapes, specifically turbine blades, from data with unknown topology

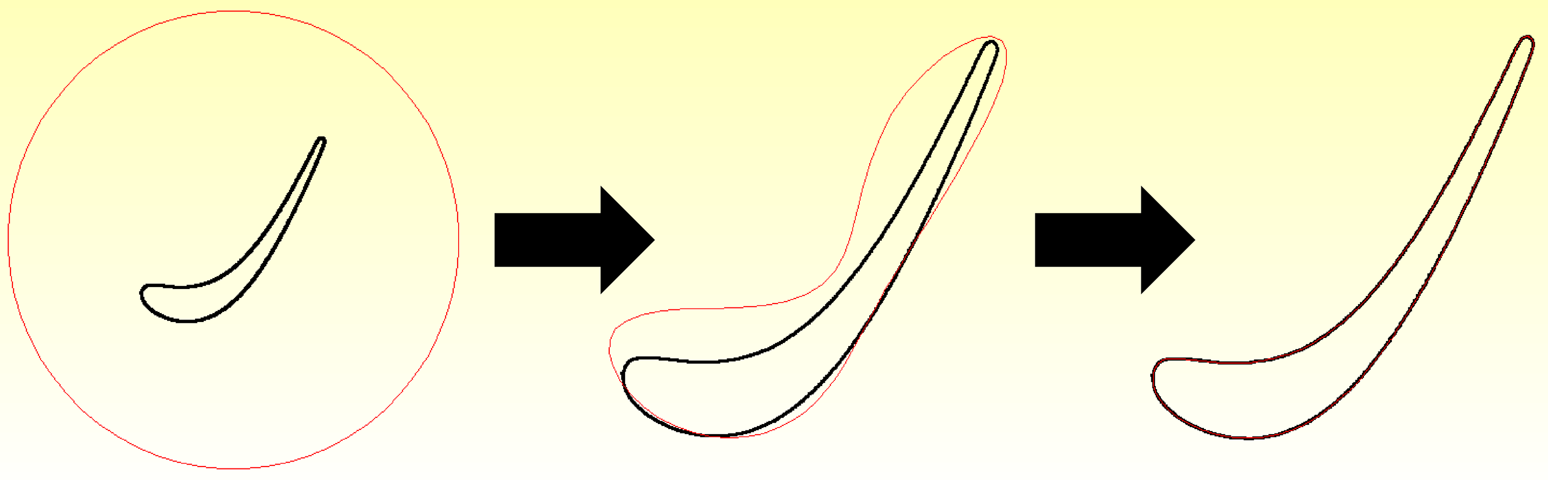
Objective function

$$\min_{t_1, t_2, \dots, t_m} \sum_j \left((MT_m)_j - \delta_j \right)^2$$

M is the subdivision matrix (1)

$T = \{t_1, t_2, \dots, t_m\}$ is the control points translations vector

Discrete B-Spline Convection scheme



✓ NO initial parameterization
✓ NO differential calculations
✓ NO sampling requirements

✓ Invariance of final control polygon geometry to initial position and orientation

Methodology

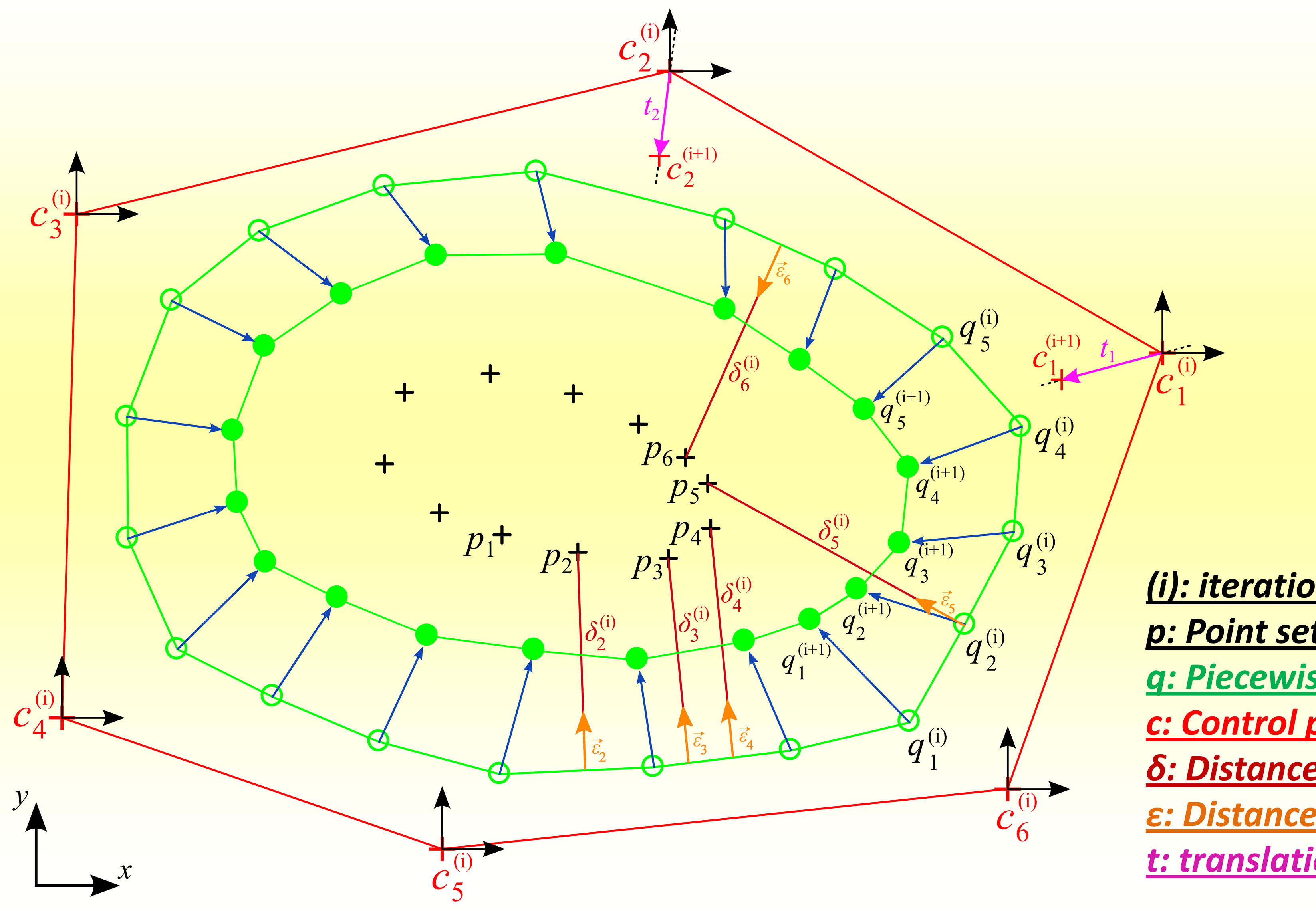
Coincide new B-Spline curve at iteration (i+1) with data points by minimizing the distances

The B-Spline (green) is initialized by a few control points around the data.

Distances δ_i are calculated with geometrical and topological considerations.

L_2 minimization → translation vectors $\{t_1, t_2, \dots, t_m\}$ by which control points must move.

If the minimization does not meet the error tolerance, point insertion is applied locally.



(1) Subdivision relation

$$q_j^{(i)} = MC_m^{(i)}$$

(2) Convection equation

$$C_m^{(i+1)} = C_m^{(i)} + T_m^{(i)}$$

(3) Solution equation

$$q_j^{(i+1)} = M\left(C_m^{(i)} + T_m^{(i)}\right)$$

(i): iteration i
p: Point set
q: Piecewise linear B-Spline curve
c: Control points
δ: Distances
ε: Distance vectors
t: translation vectors

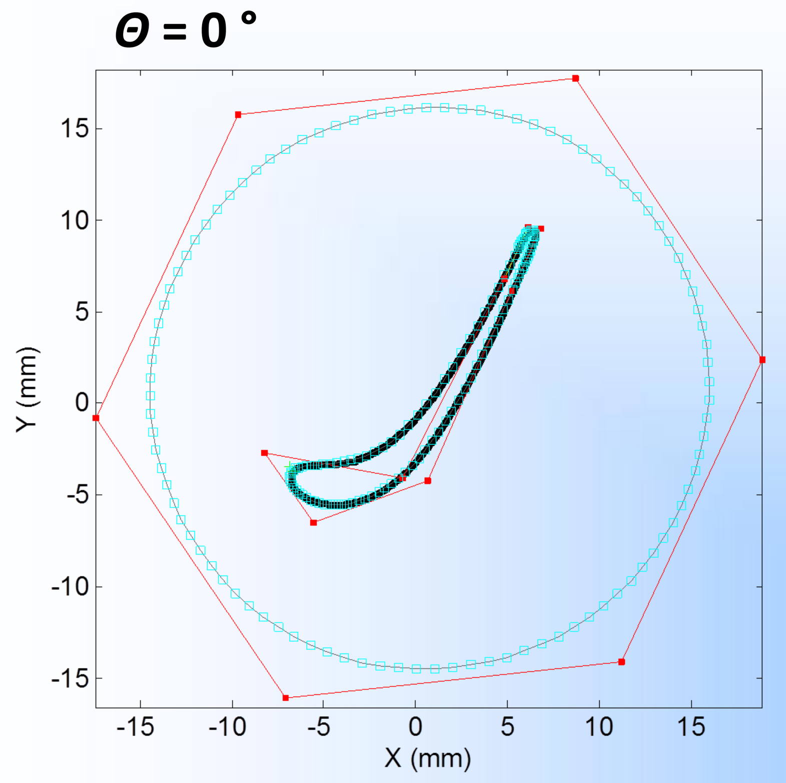
Experimental results

Invariance to point-set orientation

ε = mean of residual errors

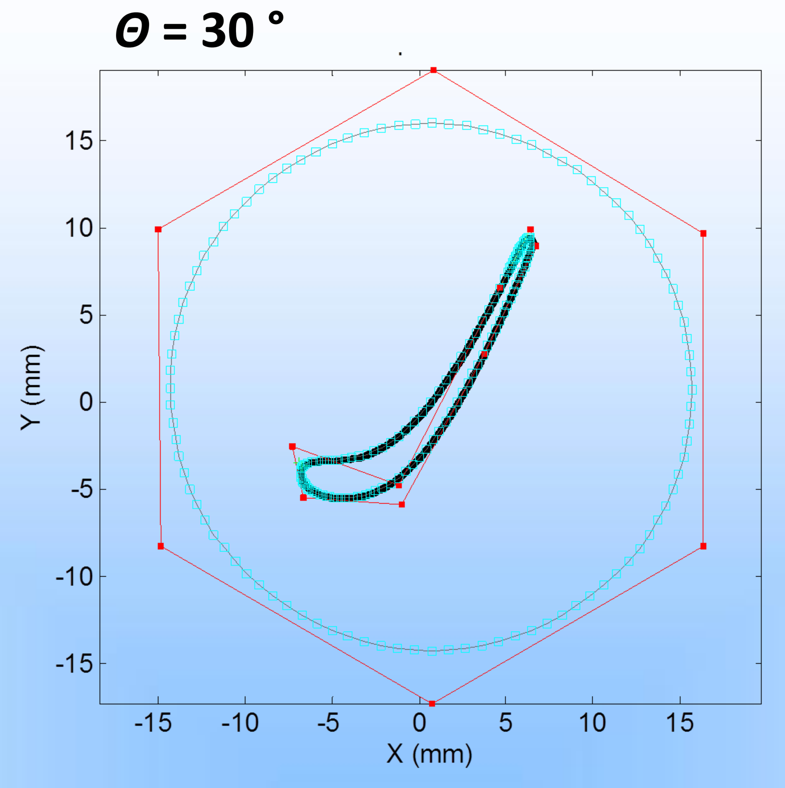
1200 points — 18 seconds

Θ = 0 °



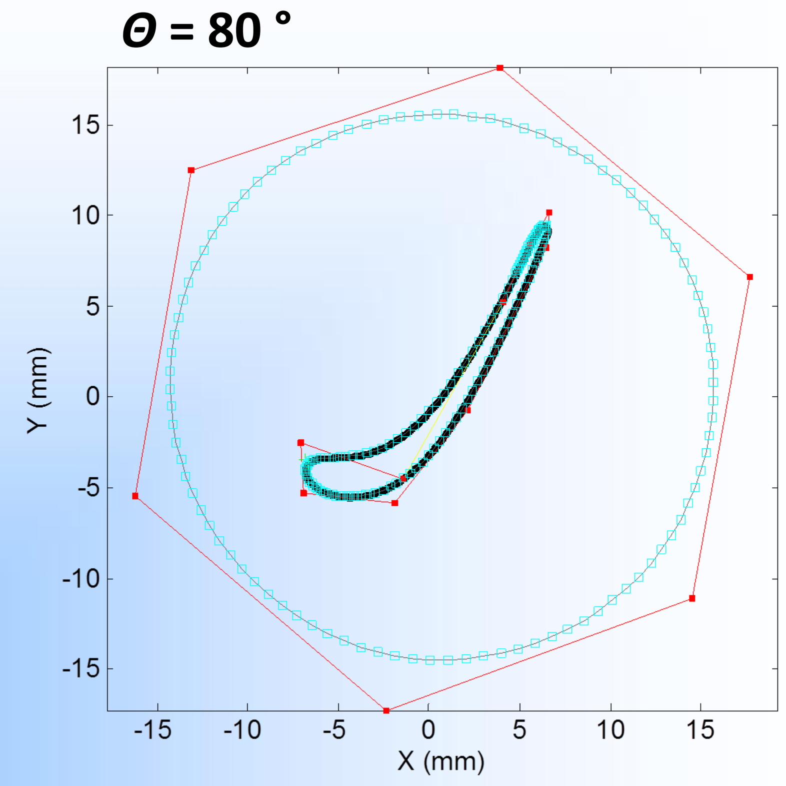
8 final control points:
ε≈0.0015 mm, 140 iterations

Θ = 30 °



8 final control points:
ε≈0.00088 mm, 140 iterations

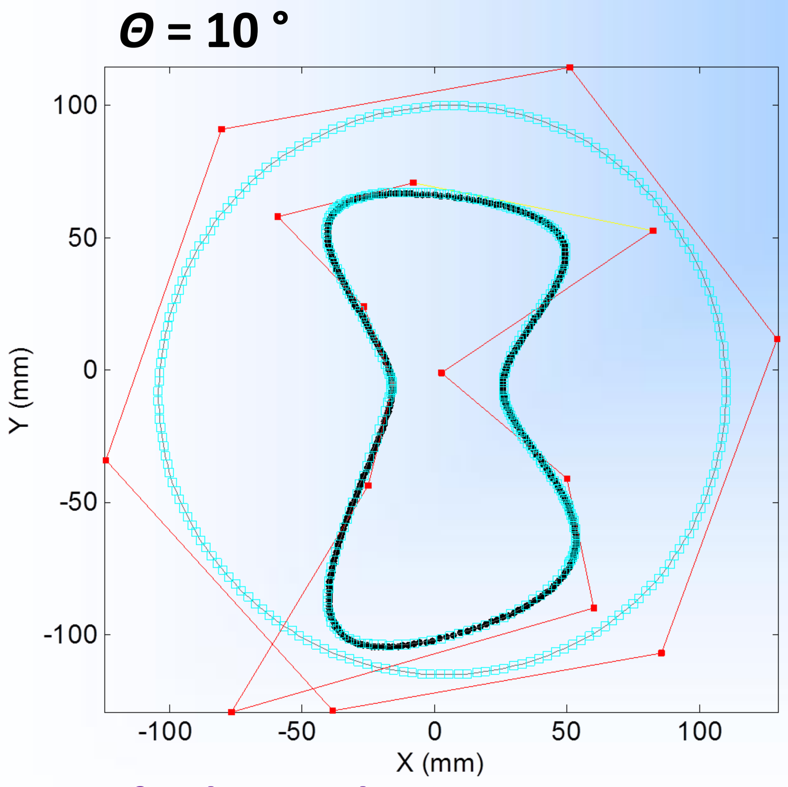
Θ = 80 °



8 final control points:
ε≈0.0023 mm, 140 iterations

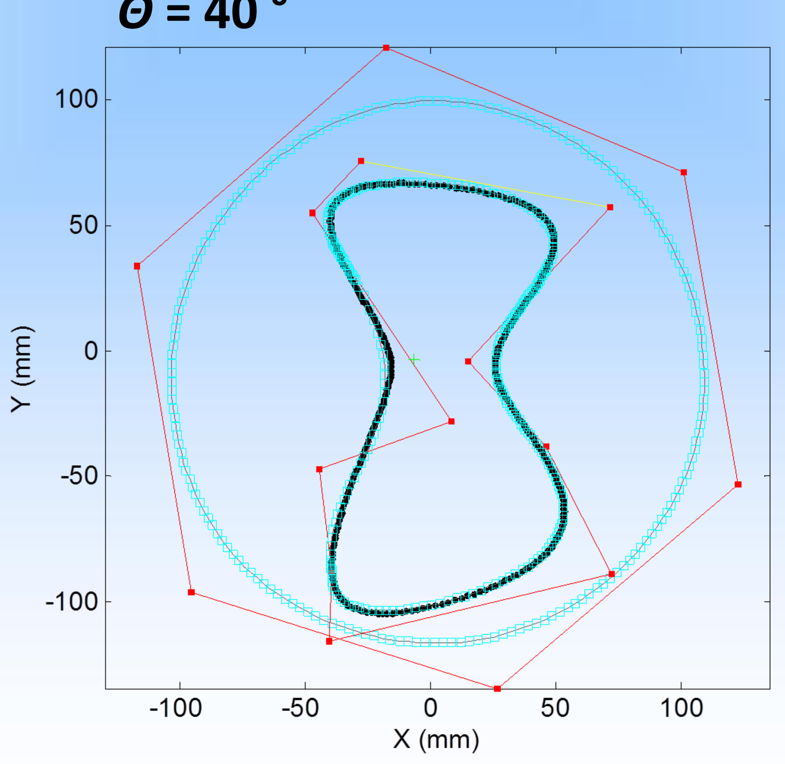
400 points — 12 seconds

Θ = 10 °



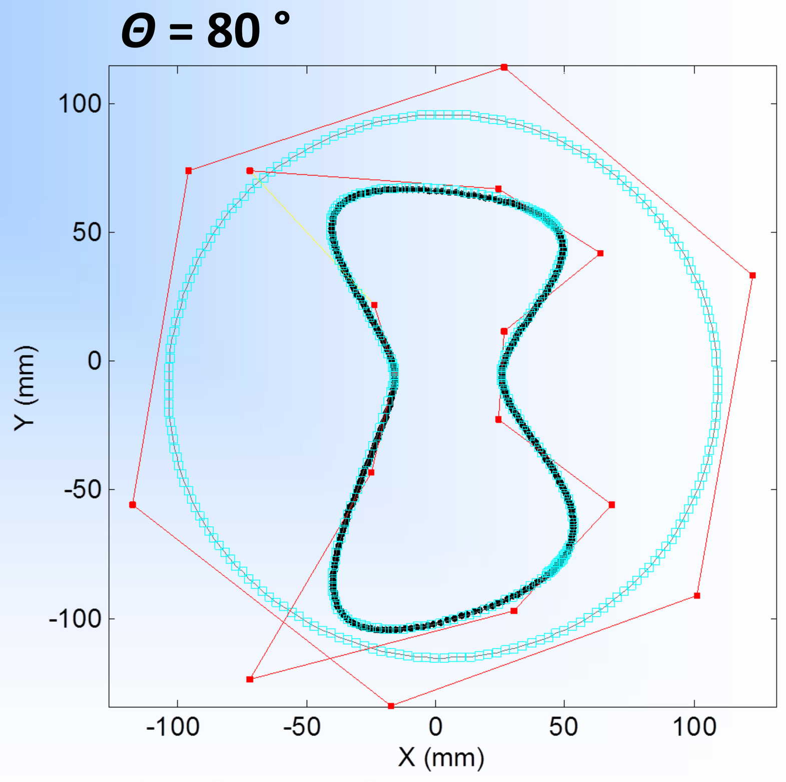
10 final control points:
ε≈0.0024 mm, 140 iterations

Θ = 40 °



10 final control points:
ε≈0.0017 mm, 140 iterations

Θ = 80 °



11 final control points:
ε≈0.00091 mm, 140 iterations

Conclusions

- The B-Spline convection algorithm is founded on discrete computations.
- The algorithm is robust regarding the relative initial position of both the B-Spline and the data.
- The algorithm is tested on several shapes and returns residual errors below threshold if not too small.
- The initial number of control points must be minimal.
- The algorithm can be subject to time complexity improvement.
- Precision is not yet controllably achievable.

[1] *Speer T., M. Kuppe, and J. Hoschek. Global reparametrization for curve approximation, in Computer Aided Geometric Design,1998.*
[2] *Wang W., H. Pottmann, and Y. Liu. Fitting B-Spline Curves to Point Clouds by Curvature-Based Squared Distance Minimization, in ACM Transactions on Graphics, 2006.*
[3] *Zheng W., P. Bo, Y. Liu, and W. Wang. Fast B-spline curve fitting by L-BFGS, in Computer Aided Geometric Design, 2012.*

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